

Ultimate charge sensitivity and efficiency of a quantum point contact with a superposed input state

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Abstract

We address the ultimate charge detection scheme with a quantum point contact. It is shown that a *superposed input state* is necessary to exploit the full sensitivity of a quantum point contact detector. The coherence of the input state provides an improvement in charge sensitivity, and this improvement is a result of the fundamental property of the scattering matrix. Further, a quantum-limited (maximally efficient) detection is possible by controlling the interference between the two output waves. Our scheme provides the ultimate sensitivity and efficiency of charge detection with a generic quantum point contact.

Detection of single electrons [1–4] is an essential ingredient for realizing quantum information processing with charge qubits. A quantum point contact (QPC) is widely used as a charge detector, with a sensitivity that extends down to the level of single electrons. It also plays an important role in investigating fundamental issues in quantum theory, such as quantum mechanical complementarity [5, 6]. It has been well understood that the phase, as well as the transmission probability, of a QPC can be utilized for charge sensing [7, 8]. The sensitivity can be described in terms of the controlled dephasing rate of the qubit induced by the interaction with the QPC detector. This is because the dephasing rate is equivalent to the rate of the (qubit) information transfer to the detector. The dephasing rate is a function of the two independent variables, ΔT and $\Delta\phi$, the sensitivities of the transmission probability and of the phase difference between the transmitted and the reflected waves, respectively [7, 9–12]. Further, it has been shown that the phase-sensitive term is dominant in a generic QPC which reduces considerably the efficiency of charge detection (typically, below 5%) [6, 13, 14]. In this context, utilizing the phase degree of freedom is important and useful for a quantum information architecture.

On the other hand, it is worth considering the following unnoticed, but fundamental, property of a QPC in the context of charge sensing. A single channel QPC is described by a 2×2 scattering matrix, which has $SU(2)$ symmetry. Neglecting the physically irrelevant global phase factor, the S-matrix has three independent physical variables for charge detection. However, all the existing experiments and theoretical proposals are based on utilizing only one or two (ΔT and $\Delta\phi$) of these variables [5–12]. Therefore, this provides an interesting question: can we exploit the three independent variables for charge detection?

In this Letter, we show that this is indeed possible. In addition to the well-known sensitivities ΔT and $\Delta\phi$, another hidden phase variable exists, and can indeed be used. We propose a scheme that utilizes this hidden variable (as well as the two other variables) by using a “superposed input state”. The hidden phase variable appears in the expression of the dephasing rate, if the input electron is in the state of a coherent superposition of two input ports. Naturally, it provides an advantage in high-sensitivity charge detection as well as deeper understanding of the quantum mechanical complementarity realized in a QPC detector. The setup proposed here provides the maximum sensitivity with a generic QPC. Further, we show that the system can be tuned for a quantum limited detection of the charge state.

One of the most remarkable features in quantum measurement is the trade-off between information transfer of the state of the *system* into the measurement *apparatus* and the back-action dephasing of the system [15–18]. The “potential” measurement sensitivity of a measurement apparatus is reflected in the dephasing rate induced by the apparatus. For an actual measurement, the information stored in the potential sensitivity should be transformed to an actual sensitivity. In general, the potential sensitivity may not be fully exploited in an actual measurement. The quantum-limited detection is a fully efficient measurement where the potential sensitivity is fully transformed to the actual sensitivity. For a practical quantum information processing, both the sensitivity and the efficiency are important: Dephasing rate (sensitivity) is the speed of the information transfer, and the efficiency is the ratio of the actual measurement rate to the dephasing rate [15, 16, 18, 19].

Let us consider a QPC charge detector, which monitors the state of a charge qubit (being 0 or 1) through mutual capacitive interactions (Fig. 1). Controlled dephasing induced by a charge detection can be implemented, for instance, by constructing interferometers which include a quantum dot [5, 6] or double quantum dots [8]. We assume that the QPC circuit has only a single transverse channel at zero temperature. Generalization to finite temperature and multichannel is straightforward. The interaction between the qubit and the QPC detector is described as a continuous weak measurement [12, 15]. The sensitivity of a possible measurement is encoded in the scattering matrix of the QPC, which depends on the qubit state j ($= 0$ or 1):

$$S_j = \begin{pmatrix} r_j & t'_j \\ t_j & r'_j \end{pmatrix}. \quad (1)$$

The scattering matrix transforms the input states α and β into the output γ and δ as

$$\begin{pmatrix} c_\gamma \\ c_\delta \end{pmatrix} = S_j \begin{pmatrix} c_\alpha \\ c_\beta \end{pmatrix}, \quad (2)$$

where c_l is the annihilation operator of an electron at lead l ($\in \alpha, \beta, \gamma, \delta$). For a single scattering event in the QPC detector, the initial state of the system before scattering can be represented as a product state of the two subsystems:

$$|\Psi_0\rangle = (a_0|0\rangle + a_1|1\rangle) \otimes |\chi_{in}\rangle, \quad (3)$$

where $a_0|0\rangle + a_1|1\rangle$ is the initial state of the charge qubit, and $|\chi_{in}\rangle$ is the input state of the QPC detector.

Our strategy here is to introduce a *superposed input* state from the two input sources α and β :

$$|\chi_{in}\rangle = (\sqrt{p}c_\alpha^\dagger + \sqrt{1-p}e^{i\theta}c_\beta^\dagger)|F\rangle, \quad (4)$$

instead of the conventional way of injecting the probe electrons from a single source. The parameters p and θ determine the degree of splitting and the relative phase between the two input waves, respectively. In a real experiment, these parameters can be tuned by placing another QPC, before injecting electrons into the region of the interactions. $|F\rangle$ denotes the ground state (Fermi sea) of the electrodes.

Upon a scattering, the system is entangled as

$$|\Psi\rangle = a_0|0\rangle|\chi_0\rangle + a_1|1\rangle|\chi_1\rangle, \quad (5)$$

where the output state of the QPC detector $|\chi_j\rangle$ is given by

$$|\chi_j\rangle = (\tilde{r}_j c_\gamma^\dagger + \tilde{t}_j c_\delta^\dagger)|F\rangle, \quad (6a)$$

$$\tilde{r}_j = \sqrt{p}r_j + \sqrt{1-p}e^{i\theta}t'_j, \quad (6b)$$

$$\tilde{t}_j = \sqrt{p}t_j + \sqrt{1-p}e^{i\theta}r'_j. \quad (6c)$$

Charge sensitivity is reflected in the reduced density matrix of the qubit, $\rho = \text{Tr}_{\text{QPC}}\{|\Psi\rangle\langle\Psi|\}$. Upon a single scattering event, its off-diagonal element ρ_{01} is reduced ($\rho_{01} \rightarrow \lambda\rho_{01}$) by the coherence factor λ

$$\lambda = \langle\chi_1|\chi_0\rangle. \quad (7)$$

We consider the continuous weak measurement limit, where the single scattering event provides only a slight modification of the qubit state ($\lambda \approx 1$). The scattering through the QPC takes place on a time scale much shorter than the relevant time scale in the qubit. In our particular case of a QPC with the applied bias voltage V , this corresponds to $\Delta t \ll 1/\Gamma_d$, where $\Delta t \equiv h/eV$ is the average time interval [20] between two successive scattering events, and Γ_d is the dephasing rate. In this process, the magnitude of ρ_{01} decays as

$$|\rho_{01}| = e^{-\Gamma_d \Delta t} |\rho_{01}^0| \quad (8)$$

with the dephasing rate $\Gamma_d = -\frac{\ln|\lambda|}{\Delta t}$. In a conventional scheme with single input port ($p = 0$ or $p = 1$), the dephasing rate is determined by the charge sensitivities of the two independent parameters, namely $T_j = |t_j|^2$ and $\phi_j = \arg(t_j/r_j)$. This is because the qubit

state information can be extracted either through the transmission probability (with a direct current measurement), or through the relative phase shift between the transmitted and the reflected output waves (by constructing an interferometer). On the other hand, our scheme provides an additional sensitivity on the parameter $\varphi_j \equiv \arg(t_j/r'_j)$, and the dephasing rate is given as

$$\Gamma_d = \frac{1}{\Delta t} [u_1(\Delta T)^2 + u_2(\Delta\phi)^2 + u_3(\Delta\varphi)^2 + u_4(\Delta T\Delta\phi) + u_5(\Delta T\Delta\varphi) + u_6(\Delta\phi\Delta\varphi)], \quad (9)$$

with parameter-dependent dimensionless coefficients u_i ($i = 1, 2, \dots, 6$). ΔT is the sensitivity of the transmission probability, that is, $\Delta T = |t_1|^2 - |t_0|^2$. The phase sensitivities are defined in the same way as $\Delta\phi = \phi_1 - \phi_0$ and $\Delta\varphi = \varphi_1 - \varphi_0$.

The key point of Eq. (9) is that Γ_d is a function of the three independent charge sensitivities, ΔT , $\Delta\phi$, and $\Delta\varphi$, in contrast to the well-known expression of the dephasing rate having only two sensitivities, ΔT and $\Delta\phi$ [7, 9, 12]. The physical meaning behind Eq. (9) can be understood as follows. First, a single channel QPC is in general described by a $SU(2)$ matrix which has three independent physical variables (just as in any spin-1/2 problem). The third hidden variable $\Delta\varphi$ appears due to the superposed input. Physically, φ_j is the relative phase between the two amplitudes, t_j and r'_j . These are the two amplitudes injected from the two different inputs and combined into a single output. Naturally, the sensitivity of this phase appears only by using a superposed input. In the limit of single input ($p = 0$ or $p = 1$), Eq. (9) reduces to the existing result

$$\Gamma_d \rightarrow \Gamma_d^0 = \frac{1}{\Delta t} \left[\frac{(\Delta T)^2}{8T(1-T)} + \frac{1}{2}T(1-T)(\Delta\phi)^2 \right], \quad (10)$$

where $T = (|t_0|^2 + |t_1|^2)/2$.

With the additional phase sensitivity $\Delta\varphi$, we can achieve an improvement of the overall sensitivity. In the following, we discuss how it can be done in a systematic way. For simplicity, we consider a low efficiency limit ($\Delta T \ll \Delta\phi, \Delta\varphi$), where the direct current measurement through the QPC extracts only a very small portion of the charge state information. This limit is meaningful because of the great potential for improvement of detection by controlling the interference. In addition, it has been argued [13, 14] that a generic QPC would show a low efficiency, which has also been observed experimentally with its efficiency below 5% [6].

In this limit ($\Delta T \rightarrow 0$), Γ_d of Eq. (9) is reduced to $\Gamma_d = \{u_2(\Delta\phi)^2 + u_3(\Delta\varphi)^2 + u_6(\Delta\phi\Delta\varphi)\}/\Delta t$. This value of Γ_d can be controlled by the two input parameters p and θ of the input state (Eq. (4)). It is straightforward to find that the maximum dephasing rate (maximum sensitivity)

$$\Gamma_d^M = \frac{1}{2\Delta t} \left\{ \frac{1}{4} [(\Delta\phi)^2 + (\Delta\varphi)^2] + (T - 1/2)(\Delta\phi)(\Delta\varphi) \right\} \quad (11a)$$

is achieved for the particular input state $|\chi_{in}\rangle = |\chi_{in}^M\rangle$:

$$|\chi_{in}^M\rangle = \frac{1}{\sqrt{2}}(c_\alpha^\dagger - ie^{i\varphi_0}c_\beta^\dagger)|F\rangle. \quad (11b)$$

Notably, Γ_d^M is always larger than Γ_d^0 (the dephasing rate of the qubit state when the conventional input ($p = 0$ or $p = 1$) is used): $\Gamma_d^0 = \frac{1}{2\Delta t}T(1 - T)(\Delta\phi)^2$. The amount of the sensitivity enhancement is found to be

$$\Delta\Gamma_d \equiv \Gamma_d^M - \Gamma_d^0 = \frac{1}{2\Delta t} [(2T - 1)\Delta\phi + \Delta\varphi]^2. \quad (11c)$$

That is, the sensitivity enhancement depends on the parameters $\Delta\phi$, $\Delta\varphi$ and T . Interestingly, a sensitivity enhancement is obtained even for $\Delta\varphi = 0$, where the third variable φ_j has no charge sensitivity.

Since the variables $\Delta\phi$ and $\Delta\varphi$ can be determined experimentally, a systematic improvement of the sensitivity is possible. Later we will briefly discuss how it can be done experimentally. The relation between $\Delta\phi$ and $\Delta\varphi$ is not universal but depends on the details of the qubit-QPC interaction. Here we consider a simple potential shift model [14] where an extra charge of a qubit provides a uniform shift of the potential. This model is suitable for describing the low efficiency limit of ($\Delta T \rightarrow 0$) charge detection [14]. In this model, one can find that $\Delta\varphi = -\Delta\phi$ [21]. Fig. 2 displays a plot of the dephasing rate Γ_d as a function of p and θ for this case ($\Delta\varphi = -\Delta\phi$). The maximum dephasing rate Γ_d^M is achieved for $p = 1/2$ and $\theta = \varphi_0 - \pi/2$, that is, for $|\chi_{in}\rangle = |\chi_{in}^M\rangle$, which is consistent with Eq. (11).

The setup of Fig. 1 is not enough for an actual measurement of the charge state. It can be overcome by putting a *measurement* QPC, (labeled as QPC_m), to compose an interference between the transmitted and the reflected waves (see Fig. 3). This scheme is particularly useful in the limit of low efficiency of the QPC interacting with the qubit. For a conventional input scheme of electrons ($p = 0$ or $p = 1$ limit in our setup), it has been theoretically shown in Ref. 18 that the full amount of information can be extracted (=“quantum limited detection

(QLD)”) by controlling QPC_m. In the following, we show that a QLD is also possible in our scheme, with the improved sensitivity.

With a *measurement* QPC (QPC_m), the scattering matrix of the interacting QPC, S_j , of Eq. (1) is transformed as

$$S_j \longrightarrow S^m S_j, \quad (12)$$

where

$$S^m = \begin{pmatrix} r^m & t'^m \\ t^m & r'^m \end{pmatrix} \quad (13)$$

is the scattering matrix of QPC_m.

The most interesting case is to inject the maximally sensitive input state, $|\chi_{in}^M\rangle$, of Eq. (11b). For this particular input state, the probe electron state is transformed to the output

$$|\bar{\chi}_j\rangle = (\bar{r}_j c_\gamma^\dagger + \bar{t}_j c_\delta^\dagger) |F\rangle, \quad (14a)$$

where

$$\bar{r}_j = \frac{1}{\sqrt{2}} \{r^m r_j + t'^m t_j - ie^{i\varphi_0} (r^m t'_j + t'^m r'_j)\}, \quad (14b)$$

$$\bar{t}_j = \frac{1}{\sqrt{2}} \{t^m r_j + r'^m t_j - ie^{i\varphi_0} (t^m t'_j + r'^m r'_j)\}. \quad (14c)$$

Note that the dephasing rate of Eq. (9) is invariant upon scattering at QPC_m, due to the unitarity of S^m . After passing through QPC_m, the output state $|\chi_j\rangle$ (Eq. (6a)) is transformed to $S^m |\chi_j\rangle$. However, the scalar product (λ) of the two detector states (Eq. (7)) is invariant because $\lambda \rightarrow \bar{\lambda} = \langle \chi_1 | S^{m\dagger} S^m | \chi_0 \rangle = \langle \chi_1 | \chi_0 \rangle = \lambda$.

The QLD can be achieved from the condition $\Delta\bar{\phi} \equiv \bar{\phi}_1 - \bar{\phi}_0 = 0$, where $\bar{\phi}_j = \arg \bar{t}_j / \bar{r}_j$. This is the relation that the measurement rate reaches the dephasing rate [15, 16, 18, 19]. We find that this leads to the condition

$$\Delta\bar{\phi} = \frac{1 - 2T^m}{1 - 4T^m(1 - T^m) \sin^2 \Theta} \Delta\phi = 0, \quad (15)$$

where $T^m = |t^m|^2$ and $\Theta = \arg(t^m/r'^m) - \arg(t_0/r_0)$. Therefore, the QLD can be easily achieved by tuning the transmission probability of the *measurement* QPC as

$$T^m = 1/2. \quad (16)$$

The two conditions, Eq.(11b) and Eq. (16), provide the *ultimate sensitivity and efficiency* that can be extracted from a generic single-channel QPC.

Finally, we briefly describe how this ultimate scheme of maximum sensitivity and efficiency can be experimentally realized. In practice, we need three quantum point contacts that form a double interference scheme (see Fig. 3), which is an extension of the electronic Mach-Zehnder interferometer [22]. The superposed input state is generated by QPC_{*i*} (“*input* QPC”). The maximally sensitive input state $|\chi_{in}^M\rangle$ (Eq. (11b)) can be easily prepared by controlling QPC_{*i*}. This input state is interacting with the qubit at the “*main* QPC”. The efficiency is independently controlled with QPC_{*m*}, the “*measurement* QPC”.

Further, the phase sensitivities $\Delta\phi$ and $\Delta\varphi$ (or equivalently, ϕ_j and φ_j with the two charge states $j = 0, 1$) can be measured in the setup of Fig. 3 as follows. $\phi_j = \arg t_j/r_j$ is the relative phase between the two split waves (at the *main* QPC) of a single incident wave. This can be directly achieved by injecting a conventional input state with $p = 1$ ($|\chi_{in}\rangle = c_\alpha^\dagger|F\rangle$) (or with $p = 0$ ($|\chi_{in}\rangle = c_\beta^\dagger|F\rangle$)). The phase ϕ_j appears in the interference pattern at the output electrode, with the condition $0 < T_m < 1$. On the other hand, $\varphi_j = \arg t_j/r'_j$ corresponds to the relative phase of the two *merged* waves initially incident from the two separated inputs α and β . This phase shift can be extracted by tuning $0 < p < 1$ and $T_m = 0$ (or $T_m = 1$). This measurement of ϕ_j and φ_j would allow a quantitative study of the controlled dephasing and measurement discussed in our proposal.

In conclusion, we have investigated the ultimate sensitivity and efficiency of a single-channel QPC as a charge detector. In contrast to the conventional charge detection schemes that utilize only one or two variables, we have shown that a QPC provides three independent physical variables for charge detection, due to the SU(2) symmetry of a scattering matrix. The hidden third information is revealed by injecting a superposed input state of the probe electrons.

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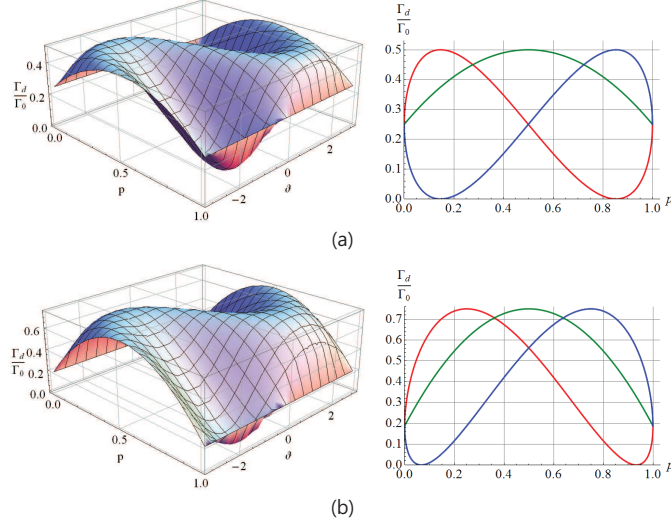


FIG. 2: Dephasing rate (Γ_d) with the potential shift model ($\Delta\varphi = -\Delta\phi$) as a function of the two input parameters, p and ϑ ($\vartheta \equiv \varphi_0 - \theta - \pi/2$) for (a) $T = \frac{1}{2}$, and for (b) $T = \frac{1}{4}$. 3D plots of the dephasing rate Γ_d (in unit of $\Gamma_0 \equiv (\Delta\varphi)^2/(2\Delta t)$) are given in the left panels. The right panels of (a) and (b) display the dephasing rate as a function of p for three different values of the input phase $\vartheta = -\pi/2$ (red), 0 (green), $\pi/2$ (blue), respectively (Color online).

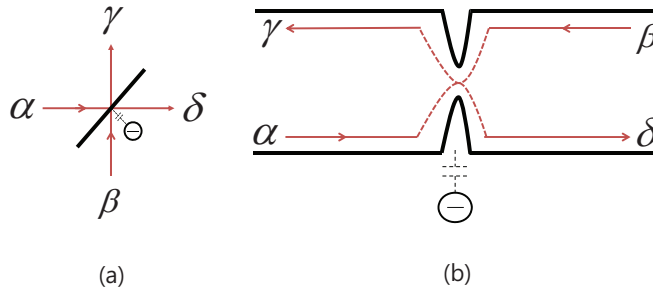


FIG. 1: (a) Charge sensing scheme of a quantum point contact with a superposed input state, and (b) a possible realization with the quantum Hall edge state (Color online).

